# Subspace Aided Parity-Based Robust Data-Driven Fault Detection in Pakistan Research Reactor-2

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## ABSTRACT

This article is concerned with FD (Fault Detection) in PARR-2 (Pakistan Research Reactor-2) using a subspace aided parity-based FD scheme. The safety is of vital importance for nuclear reactors and in time fault diagnosis is necessary for safe operation. Conventional model-based FD approaches required the mathematical model of the process. For complex systems like nuclear reactors, the modeling of the system is too much complicated. Due to the availability of huge process data of the reactor and largely inaccessibility moreover as the complexity of the process model, data-driven approaches are effective fault diagnosis techniques for reactors. Subspace aided parity-based data-driven FD approach is a simple, efficient FD approach and has required less online computations. By using a subspace-aided approach, an optimized parity vector is identified directly from the process data instead of the identification of the system model. The identified parity vector is utilized to compute residual generator that ensures robustness against system noises and disturbances and sensitivity to faults. The parity-based FD scheme is successfully implemented for PARR-2. Two possible faults in PARR-2 that are external reactivity insertion fault and control rod withdrawal fault are considered and detected successfully. GLR (Generalized Likelihood Ratio) based threshold setting is used for efficient FD and reduce false fault detection rate.

**Key Words:** Fault Detection, Data-Driven, Pakistan Research Reactor-2.

## 1. INTRODUCTION

Nuclear power plants have become predominant supporters of the energy resources of the world. They are generating about 11% of world energy IEA (International Energy Agency) [1]. Presently, almost 450 nuclear power plants are in operation and these numbers are increasing. Nuclear energy is clean, competitive, safe and reliable among other resources [2].

It is of vital importance to decrease and prevent the hazards of faults occurring within a nuclear reactor. The improvement in safety and capacity factor of a nuclear reactor is of central importance. Some certain preventive measure must be there to deal with critical issues in nuclear reactors. Three Mile Island accident [3] drew the attention of researchers towards the application of FD methods for consistent and safe operation of nuclear reactors. In that accident, the recovery procedure became complicated because of the complex alarm and indicator system. This complex system confused the reactor operational crew and they failed in recognizing the alarms and indications properly.

Model-based FD techniques are well-established and effective techniques in fault diagnosis [4-6]. These approaches depend vigorously upon accessible system model. The residual signal is constructed by comparing the output of the process and the estimated output of the analytical process model, which shows the information of fault. Model-based techniques are effective for those systems whose mathematical model is available. It is noticed that for majority of the industrial processes, modeling demands considerable engineering efforts and in some cases becomes impractical. For industrial processes, the data-driven FD approaches have been developed which do not oblige the process models a priori for developing FD systems. Over the previous decade, momentous advancement has been done in the domain of data-driven fault diagnosis [7-12]. Data-driven schemes are most appropriate for fault diagnosis of nuclear reactors, as most of the times model is unavailable and complex, meanwhile, the huge amount of input and sensor output data is available during the operation that can be used to design fault diagnosis strategy.

Among data-based FD schemes, PCA (Principal Component Analysis), FDA (Fisher Discriminant Analysis) and KFDA (Kernel Fisher Discriminant Analysis) have been successfully practiced in PARR-2 [10-11]. But these techniques involve huge online computation and also mostly faulty data is unavailable while for FDA and KFDA both healthy and faulty data are required. Remarkable work has been made by Ding et. al. [13], Wang et. al. [14], Hussain et. al. [15] and Tariq et. al. [16] in parity-based data driven approach. Nuclear reactors have a complex model and access to model is not available. Due to inaccessibility and complexity of the model, the data-driven FD schemes are well suited for nuclear reactors. Among data-driven FD schemes, parity-based data-driven FD approach is easy and simple for implementation; it also required only fault-free data for processing and less online computation that is why this approach is most suitable for FD in PARR-2.

In this article, a subspace-aided parity-based data-driven FD strategy is implemented for FD in PARR-2. The data samples are collected in fault-free conditions and under two faults i.e. external reactivity insertion and control rod withdrawal faults due to safety limitations. This fault detection approach is effective, simple and involves least online computations. It also shows robustness against disturbances and sensor noises. Generalized likelihood ratio based threshold setting is used for FD decision that also reduces the false alarm rate.

The forthcoming discussion in this article is classified as: In Section-2 review of subspace aided parity based data-driven technique is discussed then PARR-2is briefly explained in Section-3. Section-4 justifies the application of FD scheme employed in PARR-2 and its simulation results. At last, the conclusion is presented in Section-5.

## 2. REVIEW OF SUBSPACE AIDED DATA DRIVEN BASED FAULT DETECTION TECHNIQUE

Ding et. al. [13] proposed data-based parity space algorithm, then remarkable progress has been done in this direction by Wang et. al. [14], Hussain et. al. [15] and Tariq et. al. [16].

Consider a discrete LTI system given as:

$$x(k+1) = \tilde{G}x(k) + \tilde{H}(u(k) + f_a(k)) + w(k)$$

$$\tilde{y}(k) = \tilde{C}x(k) + \tilde{D}(u(k) + f_a(k)) + v(k) + f_s(k) \quad (1)$$

Here, $\tilde{G} \in R^{n\times p}, \tilde{H} \in R^{n\times \ell}, \tilde{C} \in R^{m\times n}, \tilde{D} \in R^{m\times \ell}. x(k) \in R^{n\times 1}$ is state vector, $u(k) \in R^{\ell\times 1}$,is input vector, $f_a(k)$, $f_s(k)$ are actuator and sensor fault vectors respectively and w(k), v(k) are disturbance and noise respectively.

As from (1)

$$\tilde{y}(k-1) = \tilde{C}x(k-q) + \tilde{D}(u(k-q) + f_a(k-q)) + v(k-q) + f_s(k-q) \quad (2)$$

where$q \geq 0$. Similarly

$$\tilde{y}(k-q+1) = \tilde{C}x(k-q+1) + \tilde{D}(u(k-q+1) + f_a(k-q+1)) + \quad (3)$$
$$v(k-q+1) + f_s(k-q+1)$$

$$= \tilde{C}x(k-q) + \tilde{C}\tilde{B}u(k-q) + \tilde{C}\tilde{B}f_a(k-q) + \tilde{C}w(k-q)$$
$$+ \tilde{D}(u(k-q+1) + f_a(k-q+1)) + v(k-q+1) + f_s(k-q+1) \quad (4)$$
$$\vdots$$
$$\tilde{y}(k) = \tilde{C}\tilde{G}x(k-q) + \tilde{C}\tilde{G}^{q-1}\tilde{H}u(k-q) + \tilde{C}\tilde{G}^{q-2}\tilde{H}u(k-q+1)....$$
$$+ \tilde{C}\tilde{H}u(k-1) + \tilde{C}\tilde{G}^{q-1}\tilde{H}f_a(k-q) + \tilde{C}\tilde{H}^{q-2}(k-q+1)....$$
$$+ \tilde{C}\tilde{H}f_a(k-1) + \tilde{C}\tilde{H}^{q-1}w(k-q) + \tilde{C}\tilde{G}^{q-2}w(k-q+1)...$$
$$+ \tilde{C}\tilde{H}f_a(k-q+1) + \tilde{C}\tilde{G}w(k-q) + \tilde{C}w(k-1) + \tilde{D}(u(k) + f_a(k)) + f_s(k)$$

From above expression it can be combined as:

$$\tilde{y}(k) = \Gamma_s x(k-q) + T_{uq}\left(u_q(k) f_{aq}(k)\right) + .T_{dq} w_q(k) + v_q(k) \quad (5)$$

where

$$\Gamma_q = \begin{bmatrix} \tilde{C} \\ \tilde{C}\tilde{G} \\ \vdots \\ \tilde{C}\tilde{G}^q \end{bmatrix}, u_q(k) = \begin{bmatrix} u(k-q) \\ u(k-q+1) \\ \vdots \\ u(k) \end{bmatrix}, f_{aq}(k) = \begin{bmatrix} f_a(k-q) \\ f_a(k-q+1) \\ \vdots \\ f_a(k) \end{bmatrix}$$

$$, w_q(k) \begin{bmatrix} w(k-q) \\ w(k-q+1) \\ \vdots \\ w(k) \end{bmatrix} \quad (6)$$

$$\tilde{y}_q(k) = [\tilde{y}(k-q)\tilde{y}(k-q+1)...\tilde{y}(k)]^T$$

$$T_{uq} = \begin{bmatrix} \tilde{D} & 0 & \cdots & 0 \\ \tilde{C}\tilde{H} & \tilde{D} & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \tilde{C}\tilde{G}^{q-1}\tilde{H} & \cdots & \tilde{C}\tilde{H} & \tilde{D} \end{bmatrix}, T_{dq} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \tilde{C} & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \tilde{C}\tilde{G}^{q-1}\tilde{H} & \cdots & \tilde{C} & 0 \end{bmatrix} \quad (7)$$

For subspace based data driven technique (5) could be modified as:

$$\tilde{Y}_f = \Gamma_q \tilde{X}_i + T_{uq}^i \tilde{U}_f + T_{dq}^i W_f + N_f \quad (8)$$

where,$T_{uq}^i, T_{dq}^i$ are lower block Toeplitz deterministic and stochastic matrices respectively. $W_f$ and $N_f$ are disturbance and noise block hankel matrices respectively.

Equation (8) can also be expressed in the form

$$\tilde{Z}_f = \begin{bmatrix} \tilde{Y}_f \\ \tilde{U}_f \end{bmatrix} = \begin{bmatrix} \Gamma_q & T_{uq}^i \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{X}_i \\ \tilde{U}_f \end{bmatrix} + \begin{bmatrix} T_{dq}^i W_f + N_f \\ 0 \end{bmatrix}$$

By post multiplying with $\tilde{Z}_p^T$ on both sides and dividing by N, above equation will becomes:

$$\frac{1}{N}\tilde{Z}_f\tilde{Z}_p^T = \frac{1}{N}\begin{bmatrix} \Gamma_q & T_{uq}^i \\ 0 & I \end{bmatrix}\begin{bmatrix} \tilde{X}_i \\ \tilde{U}_f \end{bmatrix}\tilde{Z}_p^T = \frac{1}{N}\Theta\tilde{Z}_p^T, \Theta = \begin{bmatrix} T_{dq}^i W_f + N_f \\ 0 \end{bmatrix}$$

Singular value decomposition of $\frac{1}{N}\tilde{Z}_f\tilde{Z}_p^T$ will be used to extract parity space $\alpha_q = \Gamma_q$ and $\Gamma_q T^i_{uq}$..

$$\frac{1}{N}\tilde{Z}_f\tilde{Z}_p^T = \tilde{U}_z E_z \tilde{V}_z, \tilde{U}_z = \begin{bmatrix} \tilde{U}_{z11} & \tilde{U}_{z12} \\ \tilde{U}_{z21} & \tilde{U}_{z22} \end{bmatrix}, \quad (9)$$

$$\Gamma_q^\perp + \tilde{U}_{z12}^T, \Gamma_q^\perp + \tilde{U}_{uq}^i = \tilde{U}_{z22}^T$$

$$I = \frac{\Gamma_q^\perp T_{uq}^i T_q^{i^T} \Gamma_q^\perp}{\Gamma_q^\perp T_{dq}^i T_{dq}^{i^T} \Gamma_q^\perp} \quad (10)$$

As an Eigen value problem, the solution of Equation (10) will be:

$$\ell_{q,\min}\left(\Gamma_q^\perp T_{dq}^i T_{dq}^{i^T} \Gamma_q^\perp - \lambda_{q,\min} \Gamma_q^\perp T_{uq}^i T_{uq}^{i^T} \Gamma_q^\perp\right) \quad (11)$$

where, $\ell_{q,\min}$ is minimum Eigen vector, $\lambda_{q,\min}$ is minimum Eigen value, and $\alpha_q = \ell_{q,\min}\Gamma_q^\perp$ is optimal robust \parity vector.

The core element of FDI is the generation of residuals$\gamma(k)$.

$$\gamma(k) \neq 0 \text{ if } f(t) \neq 0 \quad (12)$$

Residual is computed by the following relation.

$$\gamma(k) = \alpha_k\left(\tilde{y}_q(k) - T_{uq}u_q\right) \tag{13}$$

Where $\alpha_q$ is optimal robust parity vector. From Equations (5) and Equation (13) the residual generator in the existence of disturbances and noises can be composed as:

$$\gamma(k) = \alpha_q\left(T_{uq}f_{aq}(k) + T_{dq}w_q(k) + v_q(k) + f_s(k)\right) \tag{14}$$

ALGORITHM-1 SUBSPACE AIDED PARITY SPACE FAULT DETECTION

Step-1: Store n fault free input-output samples.

Step-2: Construct past and future input-output block hankel matrices and build $\tilde{Z}_p$ and $\tilde{Z}_f$.

Step-3: Perform SVD on $\frac{1}{N}\tilde{Z}_f\tilde{Z}_p^T$.

Step-4: Extract the terms $\Gamma_q^{\perp}$ and $\Gamma_{quq}^{\perp}T^i$ using Equation (9).

Step-5: Find optimal robust parity vector using Equations (10-11).

Step-6: Compute residual using Equation (13).

## 3. PAKISTAN RESEARCH REACTOR-2

PARR-2 an indigenously developed tank in pool type reactor capacity of 27-30 kW. The reactor is operating since 1991. It is a miniature neutron source reactor which operating with a 90% enriched fuel based on U-235. The fuel material consists of U-Al Alloy ($UAl_4$ – Al). Light water used for both cooling and neutron moderation purpose. Whereas heavy water, beryllium and graphite are used as neutron reflectors. The maximum thermal flux and maximum fast flux are rated as $10e^{+13}$ and $107e^{+14}$ $n/cm^2s$. It has a total number of 344 fuel rods along with control rods, 6 tie rods and 4 other dummy rods. The control rods are developed with Cd (Cadmium). The PARR-2 has been used to produce radio isotopes [17].

PARR-2 has a negative temperature coefficient due to its under- moderated core array. Therefore, the reactivity diminishes with increase in temperature, which damps the power excursions. Another salient feature of the reactor is its lower overabundance reactivity. The lower excess reactivity of core eliminates the danger of any critical incident. Thus provides additional benefits of safety. During exchange of heat between coolant and fuel an expansion in the coolant temperature is observed because the negative arbitrator coefficient of reactor minimizes the undue reactivity.

The most widely recognized faults that may happen amid the operations are control rod withdrawal and coincidental external reactivity insertion. Movement of control rod controlled the reactivity in core. Control rod withdrawal causes the insertion of positive reactivity that upgrade the power and fuel temperature. Because of inalienable safe attributes, this power outing will confine itself to 87 kW, fuel and clad temperature are beneath the immersion temperature of water. The examples are normally illuminated in the light locales amid tests. In these tests, if a fissile material case is implanted fortuitously in one among the brightening goals then reactivity is introduced in the core, state of flux can increase and power outgoing can happen that can increase the fuel and clad temperature.

## 4. APPLICATION OF FAULT DETECTION SCHEME IN PARR-2

The input-output data is acquired from accessible sensors of PARR-2 in both fault free and faulty case. Control rod withdrawal and external reactivity faults are introduced in PARR-2 and 120 measurements are recorded in presence of each fault. The inlet, outlet, pool temperatures and pool conductivity data collected from sensors. Reactivity and neutron flux are considered as actuators inputs. Observations were recorded under normal conditions in steady state with sampling time of 1 sec. The control rod was up to 15% to introduce control rod withdrawal fault and external reactivity was inserted for addition of external reactivity fault. Under each fault condition 120

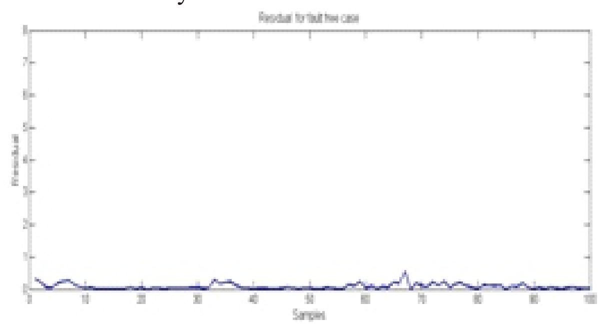

FIG. 1. RESIDUAL FOR NON-FAULTY CASE

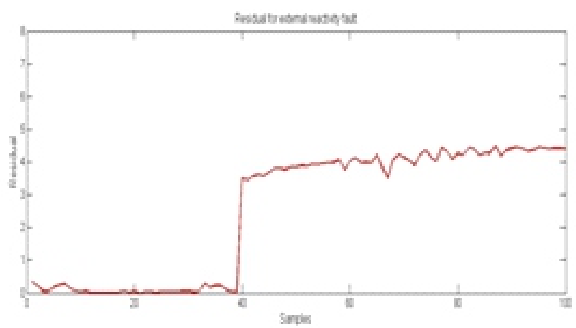

FIG. 2. RESIDUAL FOR EXTERNAL REACTIVITY FAULT

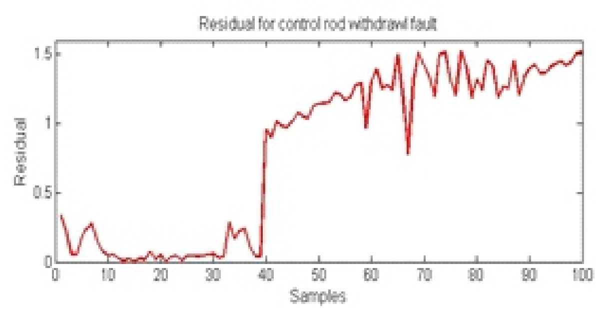


FIG. 3. RESIDUAL FOR CONTROL ROD WITHDRAWAL FAULT

measurements were recorded. The data of PARR-2 is taken from our references [10,17].

$$\widetilde{Y} \in R^{4\times 120}, \widetilde{U} \in R^{2\times 120}, \widetilde{Y}_p, \widetilde{Y}_f \in R^{24\times 109} \text{ and } \widetilde{U}_p \widetilde{U}_f \in R^{12\times 109}$$

Fig. 1 indicates the residual in non-faulty case. Then external reactivity fault is inserted from sample 40 to onward, Fig. 2 indicates the presence of external reactivity fault at sample 40. Fig. 3 shows the residual when control rod withdrawal fault is introduced at sample 40.

For successful FD and reduced false FD rate threshold is used. Residual is also effected by noise and disturbances that's why threshold setting is important for successful FD. Fault will occur if residual exceed the threshold level. GLRbased threshold setting is utilized here. GLR is explained in [4,18]. Steps for GLR based threshold setting for false detection rate<α and noise is assumed to be normal distributed $N(0,\sigma^2_r)$ is mentioned in Algorithm-2.

ALGORITHM-2 GLR BASED THRESHOLD SETTING [4]

Step-1: Find $X_a$ .e. $P[X^2>X_a] = \alpha$ using chi-square distribution.

Step-2: Set threshold as: $J_{TH} = H_a/2$..

Step-3: The testing statistic is computed as: $J = \frac{1}{2\sigma_e^2 N}\left(\sum_{i=1}^{N}\gamma(i)^2\right)$ where, γ(i)is residual of i[th] sample.

Step-4: Fault will be occurred if $J > J_{TH}$..

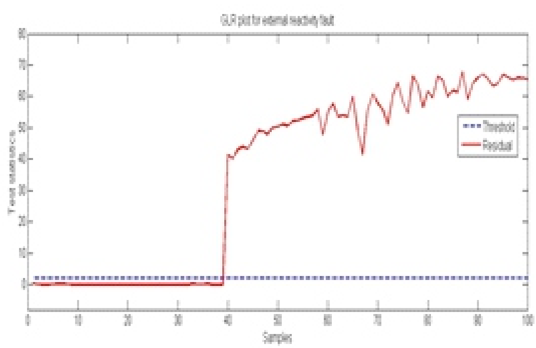


FIG. 4. GLR PLOT FOR EXTERNAL REACTIVITY FAULT

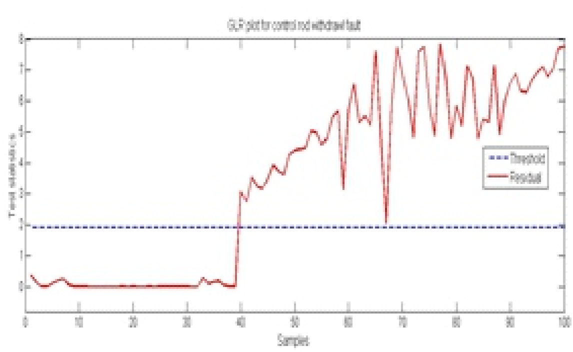


FIG. 5. GLR PLOT FOR CONTROL ROD WITHDRAWAL FAULT

GLR plots are presented in Figs. 4-5. Fig.4 indicates the residual plot in the presence of external reactivity fault and Fig.5 indicates the residual in the presence of control rod withdrawal fault. In the case of both faults, it indicates that when a fault occurs at sample 40 the residual across the threshold level. External reactivity and control rod withdrawal faults both are successfully detected. There are no false fault detection or miss fault detection as shown in Fig. 4-5.

Compared with the results of [10] the false alarm rate using this technique is reduced. The main advantage of using parity-based approach is it reduces online computation. For preprocessing faulty data is not mandatory as required in FDA, and KFDA. Performance index used to diminish the influences of disturbances and sensor noises on residual that increases the effectiveness of algorithm and becomes robust against disturbances.

## 5. CONCLUSION

In this article, the subspace-aided data-driven FD technique is successfully applied in PARR-2. The more possible faults i.e. external reactivity and control rod withdrawal faults are introduced and tested using this FD approach. Subspace aided parity-based FD technique is effective and simple in implementation for FD in PARR-2. It reduces the online computations as compared to PCA, FDA, and KFDA. It depends only on fault-free process data and information about the system model is not required. To address the issue of disturbances and sensor noise, an optimal parity vector is identified that minimizes the effect of disturbances and enhances faults effects on residual. The results demonstrate the effectiveness of the technique for PAAR-2.

## 6. FUTURE WORK

In future, the work can be extended to identify the level of fault so that it can be tolerated as much as it is possible.

## ACKNOWLEDGEMENT

The research work is supported by Pakistan Institute of Engineering & Applied Sciences, and Higher Education Commission, Islamabad, Pakistan.

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