Fixed order robust Controller Design by using H_∞ Loop Shaping and Immune Algorithm for Ball and Hoop System

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Abstract

This work presents an innovative design practice for determining the fixed order robust proportional-integral-derivative (PID) controller for ball and hoop system using the immune algorithm (IA). The paper demonstrates how to make use of the IA to search the optimal PID-controller gains. This approach has much better characteristics, including easy to implement, sure convergence attribute and fine computational efficacy. The optimum PID-controller tuning yields high-class solution. To support the predicted performance of the proposed IA based scheme a performance criterion i.e. cost function is also defined, and the preferred practice was more proficient and robust in getting better step response of ball and hoop system.

The simulation results demonstrate that IA- based PID controller be able to compensate the effect and improve the performance of control system. Additionally, the proposed design practice overcomes the weakness of conventional practices and improvement has been accomplished in terms of time domain performance.

Keywords

PID controller; optimization; immune algorithm and cost function.

1. Introduction

In recent times, industrial process control techniques have made great progress. Various control techniques have been developed such as adaptive control, neural control, and fuzzy control [1-2]. Amongst them, the top recognized is the proportional-integral-derivative (PID) controller, which has been widely used in the process industry for the reason that it holds simple structure and robustness in performance in wide range of operating conditions [3].

Regrettably, it became relatively hard to tune PID controller gains since several industrial plants are often hampered with problems like high order and time delay [4]. Several techniques have been proposed for the tuning of PID controller gains. The first method used the classical tuning rule proposed by Ziegler and Nichols. Mostly, which is safe to find out optimal or near optimal PID gains with Ziegler-Nichols for several industrial plants [5].

To design a controller means select the proper gains. The major point to note is that if calculated values of gains are too large, the response will fluctuate with high frequencies. On the other hand, having too small gains would mean longer settling time. Consequently, finding the best possible values gain is a significant concern in a controller design [6]. In general, the controller design practice is iterative among controller design and cost function (CF)[1] appraisal [7].

The design of controller to stabilize complex plant and to achieve specific performance is became an open problem. The researchers proposed approaches to make simpler the controller design practices. While alternative is to minimize the closed loop CF. But, there are certain difficulties essential to the fixed order robust controller design, such as to compute the best optimal value of controller gains and minimization of (CF) [8].

The fixed order robust controllers can be achieved by using H_∞ loop shaping procedure (LSP). The drawback of this design practice is the order of controller cannot be fixed a priori. The typical requirements are: little settling time, little overshoot and minimal value of CF [9].

Recent studies have proposed an IA to resolve optimization problems in the field of control systems and computer sciences [10]. The use of IA in optimization problems have been engorged owing its significance, capability in terms of implementation and robustness to perturbation.

An IA based PID controller was designed to improve the time domain performance of ball and hoop system. The IA will be used to determine the optimal controller gains [*kp*, *ki*, kd], and minimize the CF so that the controlled system could obtain good performance and robustness.

1.1 Original Plant

The original plant is given in Eq.1 has been used in [6, 7]. Ball and Hoop system, fourth order with the transfer function as given in Eq.1

$$G(s) = \frac{1}{(S^4 + 6S^3 + 11S^2 + 6S)} \qquad (1)$$

Fig.1 shows the pole zero plot of plant Eq.1. The four real poles are S=0, S=-1, S=-2 and S=-3, system is stable.

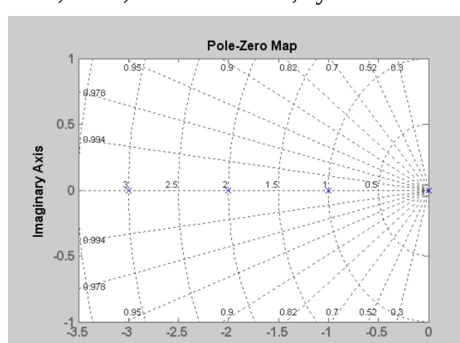

FIG. 1 SHOWS THE POLE ZERO PLOT OF NOMINAL PLANT

1 measure of performance

Perturbed plant

The perturbation to the original system transfer function has been measured in percentage. The plant poles are perturbed by 5% of the original value. Generally, perturbation in small percentage will not shift the poles in right hand side. If that is the case the plant is first needed to be stabilized by an additional local loop and then the proposed algorithm can be applied.

The plant parameters have been perturbed by 5% of the original value. The resultants transfer function is given in Eq. (2)

$$G(s)=\frac{1}{(S^4+6.3\,S^3+12.127\,S^2+6.9457\,S)} \quad (2)$$

Fig.2 shows the pole zero plot of plant Eq. (2). The four perturbed poles are S=0, S=-1.0500, S=-2.100 and S=-3.1500 while system remains stable.

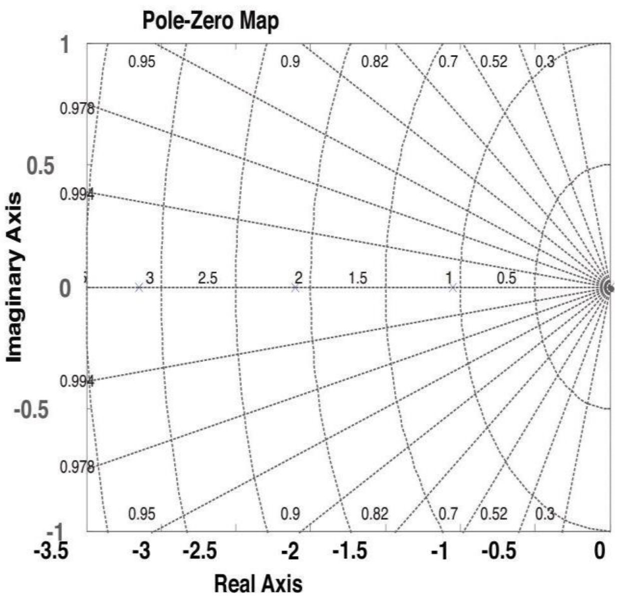

FIG. 2 SHOWS THE POLE ZERO PLOT OF PERTURBED PLANT

The paper is arranged as follows: Desired performance specifications are given in Section 2, A brief overview of *H∞* control design is presented in section 3, H_∞ loop shaping procedure is discussed in Section 4, Section 5 gives brief overview of immune algorithm, the deign aspects of IA based procedure is presented in Section 6, Section 7 presents experimental results and the conclusions are summarized in Section 8.

2. Desired Performance Specification

The main purpose of control system design is to provide good time domain performance of the controlled system. The best possible controller has to be designed such that the desired time domain performance specifications are meeting up. The desired specifications are given in Table.1

TABLE 1 DESIRED PERFORMANCE SPECIFICATION

H_∞- norm	≤ 2
Settling time	≤ 2 sec.
Rise time	≤ 1 sec
Stability margin	≤ 1
Steady state error	1

3. The H_∞ Control Design

Consider a system *P(s)* of Fg.3, with inputs *w* and outputs *z* measurement *y* control *u* and controller *K(s).* If *P(s)* is used to devise a design problem, then it will also incorporate weighs [9].

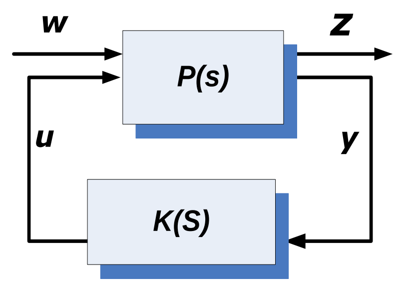

FIG.3 GENERAL H ∞ CONFIGURATION [8]

For minimizing the *H ∞*-norm of the transfer function from *w* to *z*, *P(s)* may be partitioned as given in Eq. [3]:

$$P(s)=\begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \quad (3)$$

The closed loop transfer function from *w* to *z* can be obtained directly as given in Eq. [4]:

$$Z=F_l(P,K)w \quad (4)$$

Where, $F_l(P,K)=P_{11}+P_{12}K(I-P_{22}K)^{-1}P_{21}$ is called the lower fractional transformation of P and K. Therefore, the optimal H_∞ control problem is to minimize the H_∞ norm of $F_l(P,K)$, i.e, $\|F_l(P,K)\|_\infty$

4. The H∞ Loop Shaping Procedure

H_∞ loop shaping procedure (LSP) is an efficient method used for robust controllers design and has been efficiently used in a variety of applications. Two main phases are implicated in LSP [12].

In first phase the singular values of original plant are shaped by choosing proper weights W_1 and W_2. The original plant G_0 and weights are multiplied to form a shaped plant G_s as shown in Fig. [4]. The weighs can be chosen as:

$$W_1=K_w\frac{s+\alpha}{s+\beta} \quad (5)$$

Where K_w, α, β are positive integers, β is selected as smallest number (<< 1).

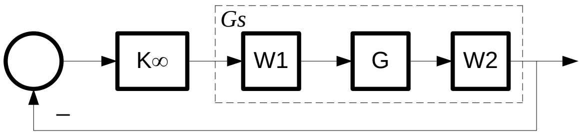

FIG. 4 BLOCK DIAGRAM OF SHAPED PLANT

In second phase the controller K_∞ is synthesized and stability margin is computed. The final controller is constructed by multiplying K_∞ with weights W_1 and W_2 as given in Eq. (6) and depicted in Fig. 5.

$$K(s)_{final} = W_1 K_\infty W_2 \tag{6}$$

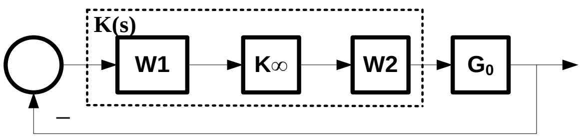

FIG.5 BLOCK DIAGRAM OF FINAL CONTROLLER

This step by step method has its groundwork in [10, 12]. After achieving the desired loop shape, H_∞-norm is minimized to find the overall stabilizing controller $K(s)_{final}$

4.1 PID Controller Background

The structure of PID controllers is very simple it works in a closed-loop system as given in Fig.6; the controller operates on the error signal that is the difference between the desired output and the actual output, and generates the actuating signal *(u)* that drives the plant. The output of a PID controller, equal to the control input to the plant, in the time-domain is as given in Eq. (7)

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de}{dt} \tag{7}$$

The transfer function of a PID controller is found by taking the Laplace transform of Eq. (9).

$$K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \tag{8}$$

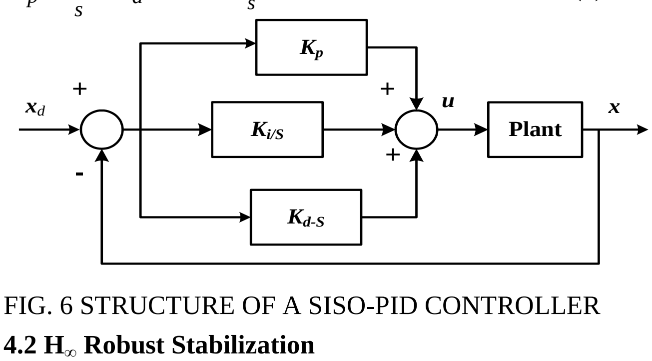

FIG. 6 STRUCTURE OF A SISO-PID CONTROLLER

4.2 H∞ Robust Stabilization

The normalized co-prime factor of the shaped plant is $G_S = W_1 G_O W_2 = NM^{-1}$, then a perturbed plant G_Δ is written as:

$$G_\Delta = (N + \Delta_N)(M + \Delta_M)^{-1} \tag{9}$$

Where, Δ_M and Δ_N are stable unknown transfer functions representing the uncertainty in the original plant Go. Satisfying $\|\Delta_M \quad \Delta_N\|_\infty \leq \varepsilon$, here $\in$ is uncertainty boundary called stability margin [13].

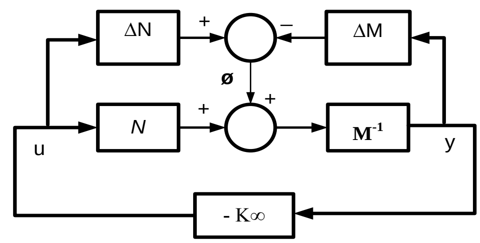

FIG.7 CO-PRIME FACTOR ROBUST STABILIZATION

Configuration shown in Fig. 7, a controller K_∞ stabilizes the original closed loop system and minimizes γ is given in Eq. (10)

$$\gamma = \underset{k}{\overset{\inf}{stab}} \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + G_s K_\infty)^{-1} M^{-1} \right\|_\infty \tag{10}$$

Where, γ is the H_∞-norm from γ to v and $(I + G_S K_\infty)^{-1}$ is the sensitivity function, the lowest achievable value of γ and correspondent maximum stability margin is computed by Eq. (11)

$$\gamma = \varepsilon_{\max}^{-1} = \sqrt{1 + \lambda_{\max}(XZ)} \tag{11}$$

Where λ_{max} denotes maximum Eigen value, Z and X are the solution to the Riccati equation [10-11]:

$$(A - BS^{-1}D^T C) + Z(A - BS^{-1}D^T C)^T - ZC^T R^{-1} CZ + BS^{-1}B^T = 0 \tag{12}$$

$$(A - BS^{-1}D^T C)^T + X(A - BS^{-1}D^T C) - XBS^{-1}B^T X + C^T R^{-1} C = 0 \tag{13}$$

Where, A, B, C, and D are state-space matrices of G, $S = I + D^T D$ and $S = I + D^T D$.

5. Overview of Immune Algorithm

An IA is a search method, starts with randomly initialization of antibodies. Then the fitness of each individual antibody is calculated. The transmission of one population to next takes place by means of immune aspects such as selection, crossover and mutation. The process chooses the fittest individual antibody from the population to continue in the next generation [2]. Moreover, an affinity is the fit of an antibody to the antigen. The role of antibody is to eliminate the antigen [9].

5.1 Modeling of gain matrix

The specified controller gain matrix consists of *n* elements:

$$\theta = \begin{bmatrix} k_1, & k_2, & k_n \end{bmatrix} \qquad (14)$$

The aim of IA is to implement heuristic search for best grouping by the these *n* elements that identify the antigen form CF Fig. 8, immune aspects includes, selection, cross over, colonial affinity and mutation are engaged to implement in the algorithm [13]

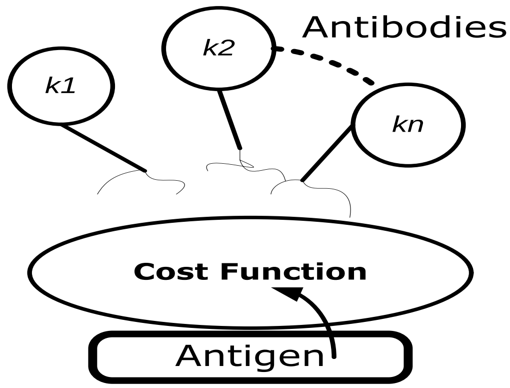

FIG.8 COST FUNCTION

6. Design Aspects of IA-PID Controller

By assuming that $K(\theta)$ is specific controller. The structure of controller has been specified previously starting the optimization procedure. The θ controller structure has been taken as vector is given by θ = [k_p, k*i*, k_d]. A set of controller parameters θ has been appraised to minimize the CF, by using Eq. (7) a controller $K(\theta)$ can be written as given in Eq. (15)

$$K(\theta) = W_1 \; K_\infty \; W_2 \qquad (15)$$

Again by assuming that W_1 and W_2 are invertible, hence,

$$K_\infty = W_2^{-1} \; K(\theta) \; W_1^{-1} \qquad (16)$$

W_2 has been selected as an identity matrix; mean that sensor noise is insignificant? By substituting Eq. (15) in Eq. (9), the H_∞-norms of the transfer functions matrix from disturbances to states, which has to be, minimized that is CF can be written as:

$$\left\| T_{zw} \right\|_\infty = \left\| \begin{bmatrix} I \\ W_1^{-1} K(\theta) \end{bmatrix} (I + G_S W_1^{-1} K(\theta)(I \quad G_S) \right\|_\infty \qquad (17)$$

5.2 Proposed approach using IA

The main steps for implementing the IA to design of robust controller are:

Step-1 calculate gamma using Eq. (11), returned variable γ is the inverse of the magnitude of uncertainty so the $\gamma \leq 4$ is requisite. If γ is > 4, it means weights are unsuitable with robust stability; the weights are to be adjusted.

Step-2 Generate initial population of antibodies as sets of parameters θ

Step-3 calculate CF of each antibody using Eq. (17) by considering θ as each string of antibodies as a vector of controller gains.

Step-4 the colonial affinity of each antibody can be calculated by using Eq. (17), best antibody in the present problem is chosen as an antigen, which has minimum CF.

$$Affinity = \frac{f(antigen)}{f(antiboby)} \qquad (18)$$

Flowchart for the above steps is depicted in Fig. 9.

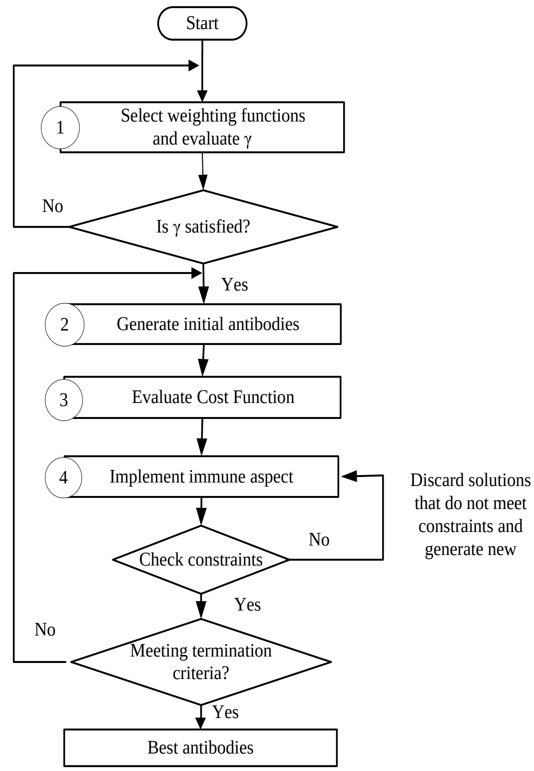

FIG. 9 FLOW CHART OF PROPOSED SCHEME

7. Simulation Results

The proposed controller and their performance evaluation criteria in time domain were implemented by MATLAB. The fixed order controller design by using IA has been set in MATLAB environment to predict performance of the proposed approach. All the simulations are performed by using MATLAB codes.

Model parameters of the nominal plant are shown in the Eq. (1) as transfer function. First, we design a controller by using LSP the weights are chosen as:

$$W_1 = \frac{0.30 + 1.0}{S + 0.001} \qquad (19)$$

Where $W_2$is the identity matrix, with these weighting functions the shaped plant is computed as?

$$G_S(s) = \frac{0.30\ S + 1}{S^4 + 11.6\ S^3 + 6.0\ S^2 + 0.06\ S} \qquad (20)$$

The stabilizing controller K_∞ is obtained by using MATLAB code is as:

$$K_\infty(s) = \frac{2.66e^{-015} S^4 + 5.32e^{-015} S^3 - 8.88e^{-016} S^2 + 0.3\,S + 1}{S^5 + 6.01\,S^4 + 11.06\,S^3 + 6.1\,S^2 + 0.06\,S}$$

(21)

By using the LSP the final controller is obtained as:

$$K(s)_f = \frac{7.99e^{-016}S^5 + 4.26e^{-015}S^4 + S^3 + 0.09S^2 + 0.6S + 1}{S^{10} + 12S^9 + 58.2S^8 + 145.2S^7 + 195.9S^6 + 135.9S^5 + 38.6S^4 + 0.7S^3 + 0.3S^2 + 6S + 1} \quad (22)$$

The controller achieved by LSP given in Eq. (22) has very complex structure and is of 10th order controller; it appears that it would be not easy to implement that controller for practical applications.

Hence, an advantage of fixed order controller design can be gained from recommended method. An IA based PID controller has been considered fixed order robust controller; k_p, k_i and k_d are parameters of the controller that would be evaluated using IA. The exact controller structure is stated in Eq. (23)

$$K(\theta) = K_p + \frac{K_i}{s} + K_d s \quad (23)$$

The Mat lab based simulations has been carried out with representation of antibodies. The size of initial population was set as 100 antibodies. Colonial affinity was computed and single bit mutation was recycled, the IA parameters are shown in Table 2, on 52nd iteration of IA the optimum values for PID gains has been accomplished.

TABLE 2 SPECIFIED PARAMETERS FOR THE IA

Parameters	**Immune Algorithm**
Initial Population of antibodies	100
Selection Type	tournament
Crossover	one point
Crossover Probability	0.80
Mutation Type	single bit mutation

As for as convergence algorithm is concerned the IA converged after 52nd iteration, and provided minimal value of CF of 1.416 Fig.10 shows the plot of convergence of CF versus iterations of IA. This fulfils the stability margin of 0.872. The calculated optimal gains of IA-based controller are presented in Eq. (24)

$$K(\theta)^* = 0.301 + \frac{0.847}{S} + 0.42S \quad (24)$$

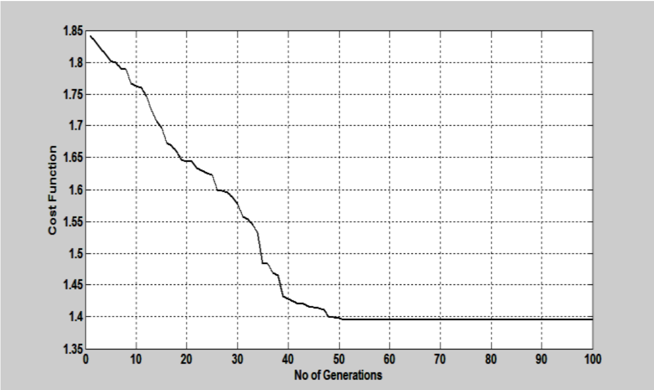

FIG. 10 CONVERGENCE OF CF VERSES ITERATIONS OF IA

The closed loop step response of the control system with IA-based controller is presented in Fig.11which presents 1.5 sec rise time, 2% overshoot, about 2 sec. settling time and zero steady state error.

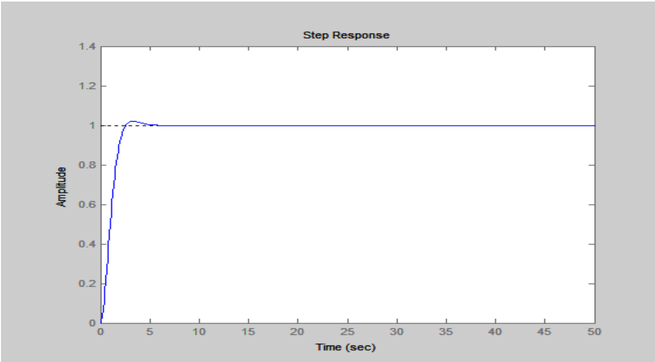

FIG. 11 CLOSED LOOP RESPONSE WITH IA CONTROLLER

7.1 Robustness Analysis

In order to validate the robustness performance of IA PID controller as given in Eq. (24) were implemented to perturbed plant Eq. (2). The closed loop step response of perturbed plant is presented in Fig.12 which presents rise time 1.5 sec., 2.2% overshoot, settling time is about 2 sec. and zero steady state error, which validates that the proposed scheme have reasonably good robustness performance.

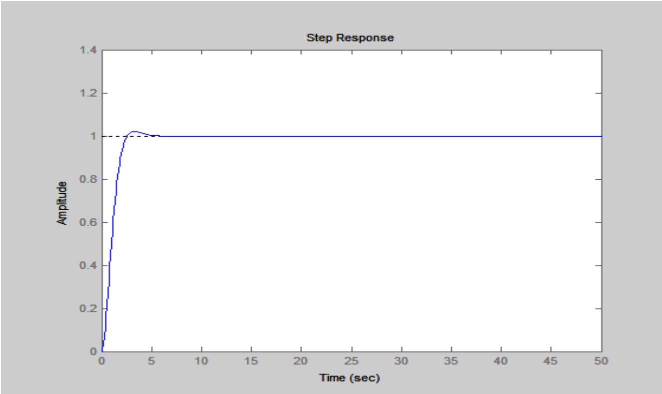

FIG. 12 ROBUSTNESS CHECK OF IA-PID CONTROLLER

8. Conclusions

In this manuscript an IA based innovative methodology has been presented. The IA has been suggested for optimization of PID controller parameter and minimization of cost function. Primary investigation demonstrates that the suggested approach can supply an optimal solution for fixed order robust PID controller.

Moreover, conventional approach used for this application experiences large settling time, large overshoot and oscillations. Henceforth, when an IA is applied to control system problems, their typical characteristics demonstrates quicker and smoother response.

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Quotations

- Don't tell other people your troubles. Half of them aren't interested, and the other half'll think you deserved it

  West African saying

- An intelligent enemy is better than an ignorant friend.

  North African saying

- The tyrant is only the slave turned inside out.

  North African saying

- If you wait for tomorrow, tomorrow comes. If you don't wait for tomorrow, tomorrow comes

  West African saying

- Rivlry is better than envy.

  Central African saying

- Hate has no medicine.

  West African saying

- Bitter truth is better than sweet falsehood.

  East African saying

- One pound of common sense requires ten pounds of common sense to apply it.

  Persian proverb

- Deal with the faults of others as gently as with you own.

  Chinese proverb

- Every good partnership is based on trust.
- Never trust a man who says, "Trust me."
- Trust is hard earned, and easily lost..
- Religions gretest miracle is the survival of faith.
- A man's faith, more than his house, is his castle.
- All are not saints that go to church.
- |Laughter is God's gift to mankind," proclaimed the preacher ponderously. "And mankind," responded the cynic, "is the proof that God has a sense of humor."
- All great deeds and all great thoughts have a ridiculous beginning. Great works are often born on a street corner or in a restaurant's revolving door.
- Think today and speak tomorrow
- Tomorrow is often the busiest day of the week.