Mixed Sensitivity H∞Controller Design for Force Tracking Control of Electro-Hydraulic Servo System

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ABSTRACT

The paper presents the mixed sensitivity H∞ controller design for force tracking control of EHSS (Electro-Hydraulic Servo System). The system is inherently nonlinear and includes hard non linearity like relationship between pressure and flow rate that effect system dynamics and deteriorate the nominal behavior of the system. To cope with such nonlinear function and improve the system performances, a mixed sensitivity H∞controller is designed using MIXSYN tool in MATLAB. The nonlinear model of EHSS is first linearized into 2nd order LTI (Linear Time Invariant) model and H∞ controller is designed. The weighting filters are included to improve the performance of system. The MATLAB function MAGSHAPE is used to tune the weight functions so that desirable gain and phase margin could be achieved. Simulation results on linear and nonlinear system show the better force tracking as compared to other robust control techniques. The proposed technique is found robust in the presence of disturbance.

Key Words: Electro-Hydraulic Servo System, Linear Time Invariant, Weighting Function, H∞Controller.

1. INTRODUCTION

EHSS is being used worldwide in several fields of engineering for position or force servo applications due to the fact that it has small volume, swift response, controlling signals flexibly, high arduousness and huge output power [1]. Due to growing demand of EHSS, the researchers paid special attention to achieve precise level in control of such system. Although, in designing the governor of EHSS several challenges appear which makes the task quite difficult. For instance, there are extremely unpredictable reasons like the flow-pressure and the dead band caused by internal outflow, hysteresis, and many more unpredictable behaviors due to linearization [2]. Hence, to the EHSM, a precise mathematical model is very tough to develop hypothetically and it is challenging for conventional PID (Proportional Integral Derivative)to provide effective control for servomechanism [3].

Enormous control techniques have been applied for position, velocity and forces/torques tracking control of EHSS.A brief review of force tracking control technique is presented in this section. The difficulties with EHSS drives are their nonlinear behavior and low damping; so the accurate control of these systems for particular applications is a difficult task to achieve [4].A layapunov based adaption scheme for force tracking control of EHSS is presented in [5-7] show worthwhile results. The proposed scheme found robust in the presence of parameter uncertainty and external disturbance.

Robust controller resides to the class of LTI system and has been studied intensively for the last two decades for position/force tracking control of EHSS [8-10]. Robust controller design preserves assurance of system performance regardless of the model inaccuracies and parameter variations. Robust controller, because of the independence on linear model of plant, guaranteed local stability.

The paper focused on special class of robust control technique in which H∞ controller is designed using mixed sensitivity approach. The proposed approach accounts non-linearity and model uncertainties in the presence of disturbance. Since the proposed approach is based upon LTI model of the system so the model is first linearized at operating point and then led for controller design.

The paper is structured as follows; Section 2 includes the calculated model for the system. Section 3 consists of the LTI model of EHSS. Section 4 discussed the design consideration of mixed sensitivity H∞ controller. Section 5 includes simulations results and finally the concluded remarks are given in section 6.

2. EHSM DISCRIPTION

The EHSM module consists of electronic drives, hydraulic actuators, and position transducers. The association between the piston position (X_p) and input voltage (u) for the servo valve is used for the development of an accurate mathematical model of EHSM servo valve [11]. The proportional valve, the asymmetric piston, controller and sensor are the main parts of control scheme. A force control servo mechanism of EHA (Electro Hydraulic Actuator) is shown in Fig. 1. The basic components of this EHSS include hydraulic supply, EHSM valve hydraulic piston, and a robust controller.

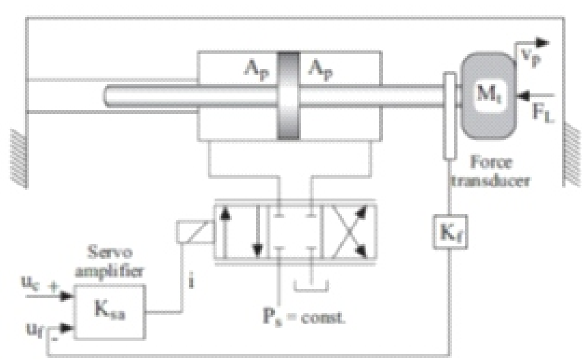


FIG. 1. EHSSFORCE TRACKING CONTROL SCHEME

2.1 System Modeling

The actual setup consists of double acting hydraulic cylinder and is connected to pressure supply thorough proportional valve. The motion of piston is due to the inflow of oil into the chamber and by controlling the oil flow in and out of the cylinder compartments its motion can be controlled. A proportional valve configuration is shown in Fig. 2. By regulating flow Q_1 and Q_2, the pressure delivered to the load can be controlled. However, the piston displacement (X_p), and the stream rates relationship rest on the changing aspects of the load applied on the piston [11].

The nonlinear mathematical representation of EHSS is model of hydraulic flow through orifice and is represented by Equation (1) as:

$$\frac{V_t}{4\beta_e}\dot{P}_L = -\dot{A}x - C_t P_L \_ Q_L \tag{1}$$

Where V_t is Actuator volume β_e is Effective Bulk Modulus, P_L is Load Pressure, x_p is Piston Position, C_t is Total Coefficient Leakage and Q_L is Load Pressure

For an ideal servo valve, the major non linearity exists between load pressure P_L and load flow Q_L, described by the relationship given in Equation (2) as:

$$Q_L = C_d w x_v \sqrt{\frac{P_s - \mathrm{sign}(x_v)P_L}{\rho}} \tag{2}$$

Where C_d is Discharge Coefficient w is Gradient Area of spool, x_v is Spool Displacement.

The spool displacement x_v is actuated through some input signal u(s) and is described by 2nd order Equation (3).

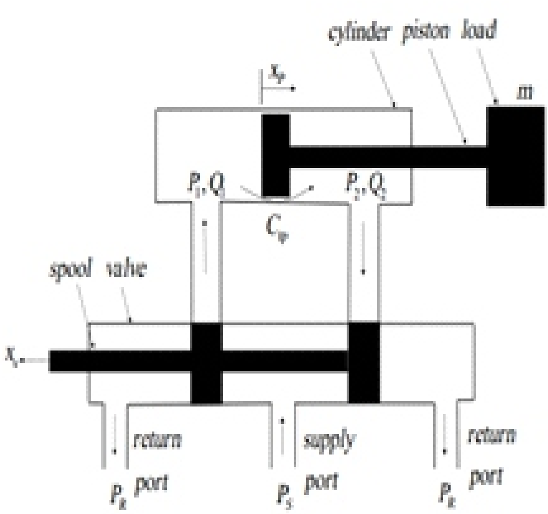


FIG. 2. EHSM FOUR-WAY VALVE CONFIGURATION

$$x_V(s)s^2 + 2\zeta\zeta_e x_v + x_v(s)\omega_e^2 = k_v u(s) \tag{3}$$

The relationship between spool displacements x_v to the input current i is also approximated to 1st order differential equation described in reference [1].

$$\tau_v \dot{x}_v = -X_v + K_v i \tag{4}$$

Where K_v and τ are gain and time constant of servo valve.

The force acting on the piston is derived by 2nd order equation,

$$F = P_L A = ms^2x(s) + Bsx(s) + K_v x(s) + F_f \tag{5}$$

P_L is the pressure delivered to the load, A is the area of cylinder, m is the mass attached to the piston and B is the damping coefficient, F_f is external disturbance load. From Equations (1-5) the system can be written in state space form as described in reference [9].

$$\dot{x}_1 = \dot{x}_2$$

$$\dot{x}_2 = \frac{1}{m}(-kx_1 - bx_2 + Ax_3)$$

$$\dot{x}_3 = \alpha x_2 - \beta x_3 + \gamma\sqrt{P_s - \mathrm{sgn}(x_4)x_3}\,x_4 \quad (6) \quad \dot{x}_4 = -\frac{1}{\tau}x_4 + \frac{K}{\tau}u$$

Where states and inputs are; x_1 is Actuator Piston position, x_2 is Actuator Piston Velocity, x_3 is Load Pressure, x_4 is Valve Position, u is Input Current to Servo Valve, and Flow rate constants are:

$$\alpha = \frac{4\beta_e A}{V_t}$$

$$\beta = \frac{4C_{tm}\beta_e}{V_t}$$

$$\gamma = \frac{4C_d\beta_e w}{V_t\sqrt{\rho}}$$

System parameters are; m is Mass of actuator, k is Spring Constant, b is Damping Constant, K is DC Gain of Servo Valve

In order to capture the key component of system dynamics, a nonlinear model of system is designed in SIMULINK/MATLAB using s-function. That model is used for validation of mixed sensitivity controller.

3. LINEAR TIME INVARIANT MODEL OF EHSS

The LTI model of the system is presented in this section prior to design of controller. As discussed earlier that nonlinear model of EHSS contained some hard non linearity that contained function sgn(x) that is not differentiable at x=0. This would make the use of any feedback linearization approach impossible.

In practice, this is equivalent to designing two controllers for the two basic models corresponding to sgn(x) = 1 and sgn(x) = -1 and switching between them according to the sgn function. This leads to a variable structure controller, and it is seldom possible to prove that, in general, the corresponding closed-loop system will not chatter [12]. In practice, this approach has been found to work correctly. It must be noted, by the way, that the same dilemma is typical of most real-life applications: the design is made under assumptions which will never be met by the implementation. For instance, feedback linearization is mostly designed assuming a continuous controller; in practice, a digital controller will be used.

In the non-reduced model (2), the variable P_L is a state variable of a subsystem with input x_v. Therefore, replacement of the nonlinear term is given as:

$$\left(\sqrt{P_s} - \mathrm{sgn}(x)x_3\right)x_4 \tag{7}$$

By applying the bilinear transformation the approximate expression becomes

$$x_4\left(P_s\frac{x_3}{2}\right) \tag{8}$$

3.1 Linear Approximation

One of the main limitations of this approximation is that the dominant hard nonlinearity of the system, which appears in the proportional valve flow Equation (2), is totally neglected. Using the above transformation, the nonlinear model, in fact, Equation (2) is approximated as:

$$\left(\sqrt{P_s} - \mathrm{sgn}(x)x_3\right)x_4 = x_4 \tag{9}$$

The state space representation of the system is then given by the following equation.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{k}{m} & -\frac{b}{m} & \frac{A}{m} & 0\\ 0 & -\alpha & -\beta & 0\\ 0 & 0 & 0 & -\frac{1}{\tau yy}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ \frac{K}{\tau}\end{bmatrix}u \tag{10}$$

$$y = \begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

Where

$$\alpha = \frac{4A\beta_e}{V_t} = 3.6\times10^{-11}$$

$$\beta = \frac{4C_{tm}\beta_e}{V_t} = 7.6\times10^{8}$$

$$\gamma = \frac{4C_d\beta_e\omega}{V_t\sqrt{P}} = 2.917\times10^{9}$$

A 2^{nd} transfer function of the EHSS with the current u(s) input to the system and force F(s) act as an output:

$$\frac{F(s)}{u} = \frac{0.0119}{S^2 + 563.33s + 1.809} \tag{11}$$

4. CONTROLLER DESING

4.1 Mixed Sensitivity H∞ Controller

This section presents the design procedure for H∞ controller using mixed sensitivity technique. Since the LTI model of the system is 2^{nd} order so the proposed controller is the sum of order of plant and weighting functions [12]. Weighting function W_1, W_2 and W_3 are introduced in the system in order to improve the stability margin and robust performance of system in the presence of parametric uncertainty [13]. The design procedure is discussed in coming sections.

4.2 Norms

H_∞ Norm for continuous time stable LTI system "G(s)" are defined using singular values of G(jw) as:

$$H_\infty \text{ norm} : \|G\|_\infty = \mathrm{Sup}\ \sigma_{i,max}\left[G(j\omega)\right]$$

For controller design, consider a CL (Closed-Loop) system shown in Fig. 3. Where r(t) is reference input, d(t) is the disturbance signal and n(t) is the measurement noise. The controlled output is define as:

$$y = Gu + d \tag{12}$$

Where

$$u = K(r-y) \tag{13}$$

Using Equation (3) in Equation (2), we can write as:

$y = GK(r-y) + d$

Or we have:

$(1 + GK)y = GK_r + d$

So controlled output

$y = (I + GK)^{-1}GKr + (I + GK)^{-1}d$

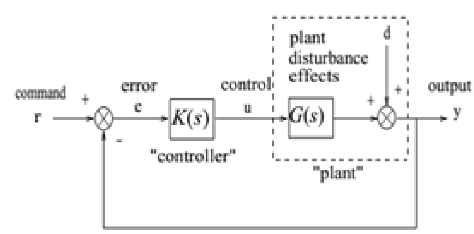


FIG.3. BLOCK DIAGRAM OF FEEDBACK CONTROL OF SISO SYSTEM

Stability performance and margins of feedback control of system given in Fig. 3, can be quantified using singular values of transfer function from "r" to outputs e, u and y that is represented by following expressions:

$$S(s) = (I + G(s) * K(s))^{-1}$$
$$R(s) = K(s)\,(I + G(s) * K(s))^{-1}$$
$$T(s)\,G(s) * K(s)\,(I + G(s) * K(s))^{-1}$$
$$T(s) + I - S(s)$$

Where S(s) is sensitivity function and T(s) is complement sensitivity function where as R(s) has no common name. S(s), T(s) and R(s) are very important for stable control system design.

4.3 Disturbance Rejection Performance

Disturbance rejection performance is determined by singular value of S(jω). Where S(jω) is transfer function of close loop from disturbance d to output y. Disturbance rejection performance is:

$$\sigma_{i,max}\left[S(jw)\right] \leq \left|W_1^{-1}(jw)\right| \tag{14}$$

$\left|W_1^{-1}(jw)\right|$ is required disturbance rejection factor

4.4 Additive and Multiplicative Robustness

Singular value bode plots for T(s) and R(s) gives measure of stability margins for feedback design of additive and multiplicative uncertainties of plants [14]. Multiplicative stability margin can be defined as the smallest stable $\Delta_m(s)$ that stabilizes system for Δ_4. System with perturbation is shown in Fig. 4.

Taking $\sigma_{i,max}\left[\Delta_m(jw)\right]$ then multiplicative robust stability margins of system can be defined by following Equation (15) as:

$$\sigma_{i,max}\left[\Delta_m(j\omega)\right] = \frac{1}{\sigma_{i,max}\left[T(jw)\right]} \tag{15}$$

Lower value of $\sigma_{i,max}\left[T(jw)\right]$ will result large $\sigma_{i,max}\left[\Delta_m(j\omega)\right]$ i.e. large stability margins. Similarly additive robust stability margins of system can be defined as:

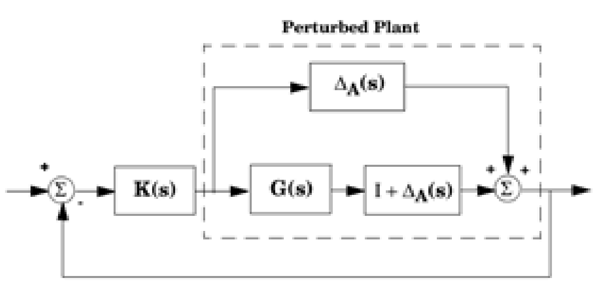


FIG. 4. UNCERTAINTIES IN SYSTEM

$$\sigma_{i,max}\left[\Delta_A(j\omega)\right] = \frac{1}{\sigma_{i,max}\left[R(jw)\right]} \tag{16}$$

From Equations (2-3) singular values based stability margin of feedback control are given as:

$$\sigma_{i,max}\left[R(jw)\right] = \leq \left|W_2^{-1}(jw)\right| \tag{17}$$

$$\sigma_{i,max}\left[T(jw)\right] = \leq \left|W_3^{-1}(jw)\right| \tag{18}$$

Where $\left|W_2^{-1}(jw)\right|$ and $\left|W_3^{-1}(jw)\right|$ is size of largest possible additive and multiplicative plant perturbation. The effects of all perturbations are combined into multiplicative perturbation. Performance specification given in Equation (14) and stability robustness specification given in Equation (17-18) are combined to get single infinity norm as:

$$\left\|T_{y1u1}\right\|_\infty \leq 1$$

T_{y1u1} is mixed-sensitivity cost function which can be written as:

$$T_{y1u1} = \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix}$$

Loop shaping is obtained for frequencies $\omega < \omega_c$ by appropriately choosing W_1 whereas $1/W_3$ targets loop shaping of frequencies $\omega > \omega_c$. Plant with weight function is shown in Fig. 5. The weighting function, according to design criteria discussed above, are tuned using MAGSHAPE command in MATLAB. A particular limitation in selection of weighting function is that it must be proper and stable.

$$W_1 = \frac{0.09s + 1}{s + 0.2}$$

$$W_2 = 0.1$$

$$W_3 = \frac{2.522}{s + 0.23}$$

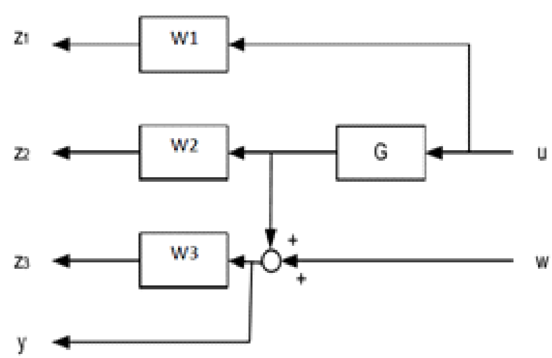


FIG. 5. AUGMENTED PLANT WITH WEIGHTS

5. SIMULATION AND RESULTS

This section includes the simulation results for force tracking control of EHSS. The designed controller satisfied the H∞ norm and quadratic H∞ performance criteria. The block diagram representation of controller implementation is shown in Fig. 6. Our design objective is fast tracking of step changes for reference inputs, with little or no overshoot. The simulation results are compared with conventional PID controller.

5.1 Simulation Results Based Upon LT1 Model of the System

Figs. 7-9 shows the simulation results based upon the LTI model of system under the step, saw tooth and sinusoidal reference inputs. It is clearly seen the performance of H∞ controller is dominant as compared to the conventional techniques of control system like PID. The performance criterion as mentioned above is the fast tracking performance under the different frequency input signals.

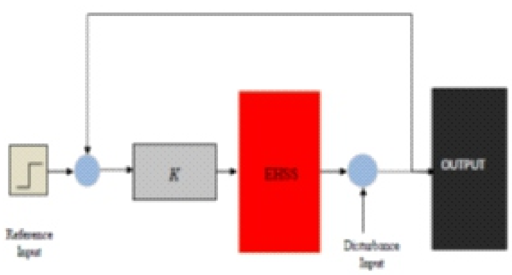


FIG. 6. BLOCK DIAGRAM OF MIX-SENSITIVITY H∞ CONTROLLER

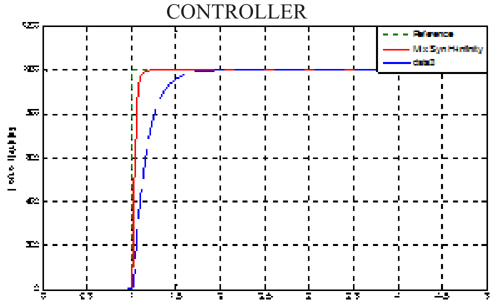


FIG. 7. FORCE TRACKING CONTROL UNDER THE STEP INPUT

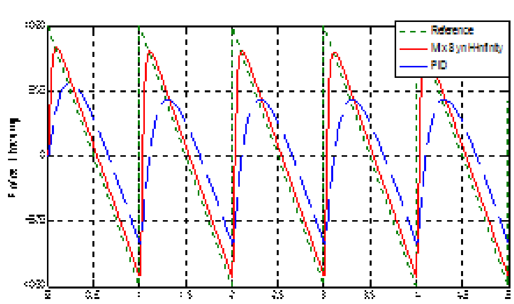


FIG. 8. FORCE TRACKING CONTROL UNDER THE SAW TOOTH INPUT

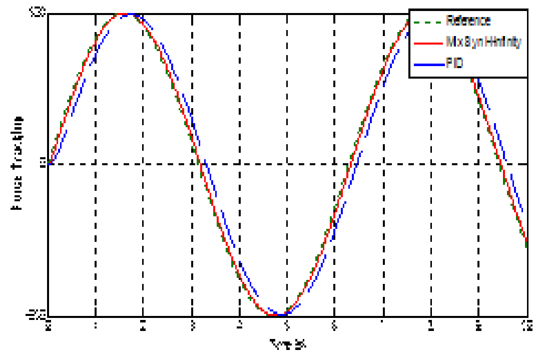


FIG. 9. FORCE TRACKING CONTROL UNDER THE SINE INPUT

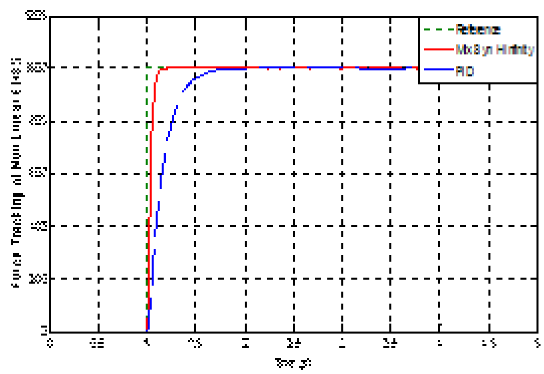


FIG. 10. FORCE TRACKING CONTROL UNDER THE STEP INPUT FOR NONLINEAR EHSS

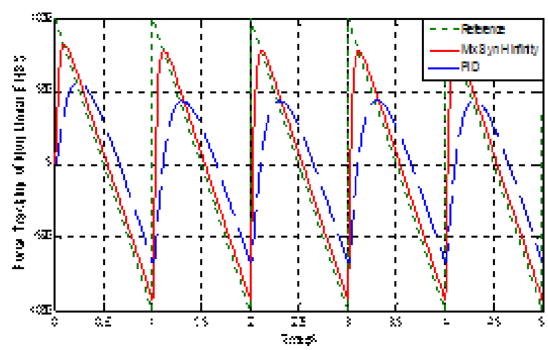


FIG 11. FORCE TRACKING CONTROL UNDER THE SAW TOOTH INPUT FOR NONLINEAR EHSS

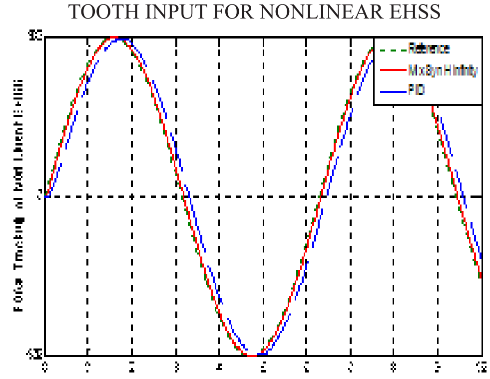


FIG. 12. FORCE TRACKING CONTROL UNDER THE SINUSOIDAL INPUT FOR NONLINEAR EHSS

5.2 Simulation Results on Non-Linear System

Figs. 10-12 shows the simulation result of force tracking control based upon the nonlinear model of the system. Simulation results on nonlinear system validate the performance of controller. Also the PID is applied on nonlinear system for comparison.

5.3 Simulation Results in the Presence of Step Disturbance

To check the robustness of the controller, a disturbance signal that is the function of state of the system is applied at the input of the nonlinear EHSS. The simulation results show the better tracking performance of mixed sensitivity Hcontroller than PID even in the presence of disturbance. Simulation result under the square and sinusoidal inputs are shown in Figs. 13-14.

6. CONCLUSION

The paper focused on mixed sensitivity H∞ pe controller design for force tracking control of EHSS. The control task is to achieve precise force tracking electro-hydraulic actuator under varying load conditions. The nonlinear system is first linearized and then H∞ controller is designed. The controller is evaluated both on linear and nonlinear model of system. Simulation results demonstrate the dominant performance of controller as compared to PID controller. Further the proposed controller is found robust in the presence of parametric uncertainty and external disturbances. The controller is simpler in design and implementation. Moreover, it also satisfied performance criteria of quadratic stability.

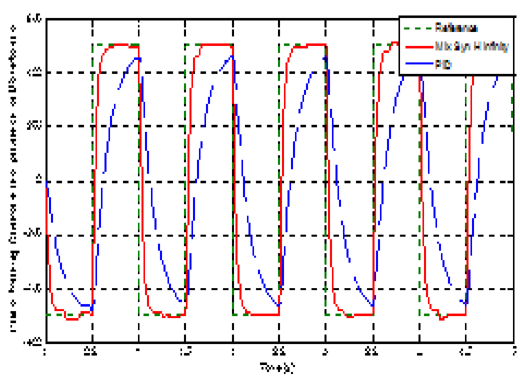
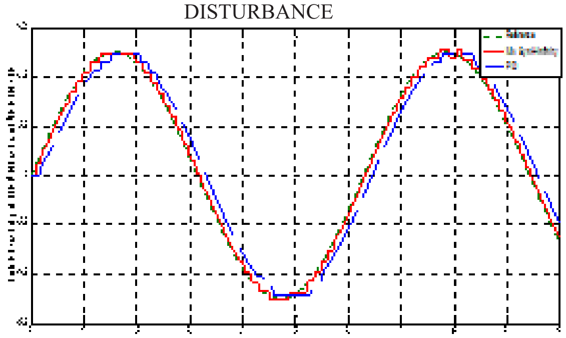


FIG 13. FORCE TRACKING CONTROL UNDER THE SQUARE INPUT FOR NONLINEAR EHSS IN THE PRESENCE OF DISTURBANCE



IG. 14. FORCE TRACKING CONTROL UNDER THE SINUSOIDAL INPUT FOR NONLINEAR EHSS IN THE PRESENCE OF DISTURBANCE

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